ИССЛЕДОВАНИЕ ГЕНЕРАЦИИ НАЙКВИСТОВСКИХ ИМПУЛЬСОВ С ПОМОЩЬЮ ГРЕБЕНКИ ОПТИЧЕСКИХ ЧАСТОТ

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По сравнению с мультиплексированием с ортогональным частотным разделением каналов передача найквистовских сигналов обладает такими преимуществами, как меньшая сложность приемников, более узкие полосы занимаемых частот и меньшее отношение пиковой к средней мощности, что обеспечивает лучшие эксплуатационные характеристики в условиях нелинейных искажений при передаче сигналов по волокну. Поскольку сигнал с sinc-огибающей во временн й области соответствует прямоугольному спектру, найквистовские импульсы могут быть получены напрямую из плоских оптических гребенок с помощью электрооптических модуляторов. Выполнен теоретический анализ генерации найквистовских импульсов с использованием трех линий гребенок и единственного модулятора интенсивности. Показано, что для генерации найквистовсих импульсов важны как амплитуды, так и фазы боковых частотных полос в гребенках оптических частот. Теоретически показано, что при наличии более чем трех линий гребенок невозможно генерировать найквистовские импульсы единственным модулятором интенсивности. Найквистовские импульсы могут быть получены использованием девяти линий с помощью двух последовательных модуляторов интенсивности, причем суммарный фазовый сдвиг, обеспечиваемый приложением постоянного напряжения к ним, должен составлять π.

Ключевые слова: *найквистовские импульсы, гребенки оптических частот, фазовые соотношения.*

INVESTIGATION ON NYQUIST PULSE GENERATION BY OPTICAL FREQUENCY COMB

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Compared with OFDM, Nyquist signal transmission has several unique advantages such as lower receiver complexity, lower receiver bandwidths and lower peak-to-average power ratios which give better performances under fiber nonlinear impairments. Since the sinc-shaped waveforms in time domain corresponds to a rectangular spectrum in the frequency domain, the Nyquist pulses can also be obtained directly from the generation of flat frequency comb with electro-optic modulators. In this paper, a complete theoretical analysis of Nyquist pulse generation with 3 comb lines generated with a single intensity modulator is developed. It is proved that both the amplitude and phase of the sidebands in the optical frequency comb are important for generation of Nyquist pulses. Theoretically it is found that it is impossible to generate Nyquist pulses with more than 3 comb lines using a single intensity modulator even if 5 flat comb lines are realized. A Nyquist pulse can be generated with 9comb lines by two cascaded intensity modulator, but the sum of the phase shift caused by the two DC voltages of the intensity modulator must be π.

Key words: *Nyquist pulses, optical frequency comb, phase relationship.*

OCIS codes: 060.2330, 060.5625, 190.4160

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Introduction

High capacity optical communication system is becoming more and more important with the ever-increasing bandwidth demand of emerging network services, including high definition video streaming, cloud computing and mobile data delivery [1–3]. Orthogonal multiplexing technologies, known as OFDM and Nyquist pulse multiplexing, are good solutions to form the data in the minimum bandwidth. Compared with OFDM, Nyquist signal transmission has several unique advantages such as lower receiver complexity, lower receiver bandwidths and lower peak-to-average power ratios which give better performances under fiber nonlinear impairments. Nyquist pulse generation can be realized by either optical or electrical schemes [4, 5] Optical Nyquist signals can provide a much higher symbol rate utilizing time domain multiplexing technique, while electrical Nyquist pulse generation is usually restricted by limited sampling rate and processing capacity of digital-to-analog convertor devices. In most recent work [6], a novel all-optical method to produce sinc-shaped Nyquist pulses of very high quality has been proposed based on the direct synthesis of a rectangular shaped and phase-locked frequency comb. A simple scheme of generating short optical pulses and flat frequency comb with an integrated dual parallel Mach-Zehnder modulator (DPMZM) driven by a single radio frequency (RF) source is proposed in [7, 8]. Only limited and specific number (3 for the intensity modulator (IM) and 5 for the DPMZM) of frequency lines can be generated in these schemes with single stage [9, 10]. But in these papers, the phase relationship between these sidebands in the generated frequency comb is seldom analyzed. The conditions for generating Nyquist pulse using a DPMZM is proposed and demonstrated in publication [11]. The amplitude and phase of the sidebands in the optical frequency comb are important for generation of Nyquist pulses. Theoretically it is found that it is impossible to generate Nyquist pulses with more than 5 comb lines using a single DP-MZM even if 7 flat comb lines are realized.

In this paper, a complete theoretical analysis of Nyquist pulse generation with 3 comb lines generated with a single IM is developed. It is proved that both the amplitude and phase of the sidebands in the optical frequency comb are important for generation of Nyquist pulses. Theoretically it is found that it is impossible to generate Nyquist pulses with more than 3 comb lines using a single IM even if 5 flat comb lines are realized.

Principle of analytical model

A general expression in the time domain for the amplitude waveform of Nyquist pulses is

$$r(t)=\frac{\sin\left(2\pi t/\tau_p\right)}{\left(2\pi t/\tau_p\right)}\frac{\cos\left(2\beta\pi t/\tau_p\right)}{1-\left(4\beta t/\tau_p\right)^2}, \qquad (1)$$

where τ_p is the pulse duration between zero crossings and β is known as a roll-off factor, which is in the range $0 < \beta < 1$. Among the class of Nyquist pulses, the sinc-shaped pulse is of particular interest owing to its rectangular spectrum and zero roll-off. So Eq. (1) can be rewritten as

$$r(t)=\frac{\sin\left(2\pi t/\tau_p\right)}{\left(2\pi t/\tau_p\right)}, \qquad (2)$$

which is a sinc-shaped pulse. In this paper, a method to generate a sequence of very high quality sinc-shaped Nyquist pulses with an almost ideal rectangular spectrum is proposed and demonstrated.

The schematic diagram of the proposed optical frequency comb generator with a rectangular shape is shown in Fig. 1. As shown in Fig. 1, a continuous wave (CW) light is launched into an IM. The output electrical field of IM can be expressed as

$$E_{out1}=\frac{E_0\exp\left(j\omega_0 t\right)}{2}\times$$
$$\times\sum_{n=-\infty}^{\infty}J_n(m)\exp\left(jn\omega_{RF}t\right)\left[\exp(j\varphi)+(-1)^n\exp(-j\varphi)\right], \qquad (3)$$

where E_0 and ω_0 are the amplitude and the angular frequency of the CW laser. $M = \pi V_{RF}/V_\pi$

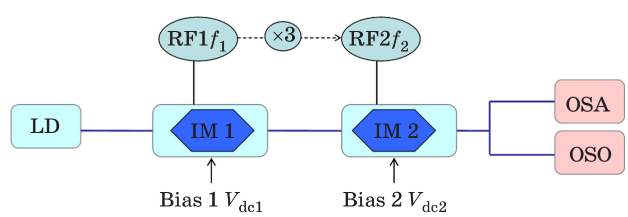

Fig. 1. The schematic diagram of Nyquist pulse. IM – intensity modulator, LD – laser diode, RF – radio frequency; DC – dc power supply; OSO – optical sampling oscilloscope; OSA – optical spectrum analyzer.

and $\varphi = \pi V_{dc}/V_\pi$ are the RF modulation index and the phase shift caused by the DC voltage. Based on $2\cos(x) = \exp(jx) + \exp(-jx)$, $2j\sin(x) = \exp(jx) + \exp(-jx)$ and $J_n(m) = (-1)^n J_n(m)$, the optical carrier, 1st-order and 2nd-order sidebands can be expressed as

$$\begin{aligned} E_0 &= \cos(\varphi) J_0(m) E_{in}(t), \\ E_{+1} &= \sin(\varphi) J_1(m) \exp\left[\left(j\omega_{RF}t + \pi/2\right)\right] E_{in}(t), \\ E_{-1} &= \sin(\varphi) J_1(m) \exp\left[-\left(j\omega_{RF}t + \pi/2\right)\right] E_{in}(t), \\ E_{+2} &= \cos(\varphi) J_2(m) \exp(j2\omega_{RF}t) E_{in}(t), \\ E_{-2} &= \cos(\varphi) J_2(m) \exp(-j2\omega_{RF}t) E_{in}(t). \end{aligned} \quad (4)$$

According to equation (4), if $|\cos(\varphi)J_0(m)| = |\sin(\varphi)J_1(m)| = A$ is true, then the optical carrier and the two 1st-order sidebands would have the same amplitude and 3 comb lines with the same amplitude are generated. When $\tan(\varphi) = +J_0(m)/J_1(m)$ is true, the output signal of IM can be expressed as

$$\begin{aligned} E_{out1} = A\Big[&\exp\left(-j\omega_{RF}t - j\pi/2\right) + \\ &+ 1 + \exp\left(j\omega_{RF}t + j\pi/2\right)\Big] E_{in}(t), \end{aligned} \quad (5)$$

which can be further rewritten as

$$\begin{aligned} E_{out1} &= \\ &= A\left[\exp\left(-j\omega_{RF}t - j\pi/2\right) + 1 + \exp\left(j\omega_{RF}t + j\pi/2\right)\right] E_{in}(t) = \\ &= A\left[\exp\left(-j2\pi f_1 t - j\pi/2\right) + 1 + \exp\left(j2\pi f_1 t + j\pi/2\right)\right] E_{in}(t) = \\ &= A\sum_{n=-1}^{1} \exp\left[2\pi jn\left(f_1 t + 1/4\right)\right] E_{in}(t) = \\ &= 3AE_{in}(t)\frac{\sin\left[3\left(\pi f_1 t + \pi/4\right)\right]}{3\sin\left(\pi f_1 t + \pi/4\right)}, \end{aligned} \quad (6)$$

where f_1 is the frequency of the RF signal, and the frequency spacing of the optical frequency comb is f_1. The frequency spacing Δf between adjacent spectral lines determines the pulse repetition period $T = 1/\Delta f$ and the rectangular bandwidth $N\Delta f$ (N being the number of lines) defines the zero-crossing pulse duration $\tau_p = 2/(N\Delta f)$. The envelope of the optical field from the IM is periodic sinc-shape and it's a Nyquist pulse train in time domain. And when $\tan(\varphi) = -J_0(m)/J_1(m)$ is true, the output signal of IM can be expressed as

$$\begin{aligned} E_{out} = A\Big[&\exp\left(-j\omega_{RF}t - j3\pi/2\right) + \\ &+ 1 + \exp\left(j\omega_{RF}t + j3\pi/2\right)\Big] E_{in}(t), \end{aligned} \quad (7)$$

which can be further rewritten as

$$\begin{aligned} E_{out} &= A\sum_{n=-1}^{1} \exp\left[2\pi jn\left(f_1 t + 3/4\right)\right] E_{in}(t) = \\ &= 3AE_{in}(t)\frac{\sin\left[3\left(\pi f_1 t + 3\pi/4\right)\right]}{3\sin\left(\pi f_1 t + 3\pi/4\right)}. \end{aligned} \quad (8)$$

If we let $|E_0| = |E_{\pm1}| = |E_{\pm2}|$, we can get

$$\begin{aligned} &J_0(m) = J_2(m), \\ &J_0(m)\left[\exp j\varphi + \exp(-j\varphi)\right] = J_1(m)\left[\exp j\varphi - \exp(-j\varphi)\right]. \end{aligned} \quad (9)$$

For example, it can be easily calculated that as $m = 1.84$ and $\varphi = 0.5$, and the optical carrier, 1st and 2nd order sidebands would have the same amplitude and 5 flat comb lines can be generated. However, it isn t a Nyquist pulse train in time domain. Equation (5) can be rewritten as

$$\begin{aligned} E_{out} &= A\Big[\exp(-j2\omega_{RF}t) + \exp\left(-j\omega_{RF}t - j\pi/2\right) + \\ &+ 1 + \exp\left(j\omega_{RF}t + j\pi/2\right) + \exp(j2\omega_{RF}t)\Big] E_{in}(t) = \\ &= A\Big[\exp(-j2\omega_{RF}t) + \exp\left(-j\omega_{RF}t - j\pi/2\right) - 1 + \\ &+ \exp\left(j\omega_{RF}t + j\pi/2\right) + \exp(j2\omega_{RF}t) + 2\Big] E_{in}(t) = \\ &= 3AE_{in}(t)\frac{\sin\left[3\left(\pi f_1 t + \pi/4\right)\right]}{3\sin\left(\pi f_1 t + \pi/4\right)} + 2AE_{in}(t). \end{aligned} \quad (10)$$

It can be seen that even the 5 frequency components have the same amplitude, but the corresponding waveforms are not sinc-shaped in the time domain.

As we know, the number of OFC can be enlarged by cascading two stages. When cascading with another IM, the optical signal with 3 spectral lines is sent to IM2, which is driven by a RF signal at f_2. To generate 9 lines OFC, the condition $f_1 = 3f_2$ or $f_2 = 3f_1$ has to be satisfied without any carrier suppression, resulting in a frequency spacing between the lines of $\Delta f = \min(f_1, f_2)$. We found that in order to make the phase of OFC is linear-locked, the phase shift difference caused by the DC voltage of the two IMs must be π. For example, when $f_2 = 3f_1$, the optical field at the output of the IM2 can be expressed as

$$\begin{aligned} E_{out2} &= A_1\sum_{n=-1}^{1} \exp\left[jn_1\left(\omega_{RF1}t + \pi/2\right)\right] \times \\ &\times A_2\sum_{n=-1}^{1} \exp\left[jn_2\left(\omega_{RF2}t + 3\pi/2\right)\right] E_{in} = \end{aligned}$$

$$= A_1 A_2 \sum_{n_2=-1}^{n_2=1} \sum_{n_1=-1}^{n_1=1} \exp\Big[j2\pi\Delta f t(n_1+3n_2)+ + j2\pi\Big(\frac{3}{4}n_2+\frac{1}{4}n_1\Big)\Big]E_{in} = = A_1A_2\sum_{n_1=-4}^{n_2=4}\exp\Big[2\pi jn\Big(\Delta ft+{}^1\!/\!_4\Big)\Big]E_{in} = = 9A_1A_2\frac{\sin\Big[9\pi\Big(\Delta ft+{}^1\!/\!_4\Big)\Big]}{9\sin\Big[\pi\Big(\Delta ft+{}^1\!/\!_4\Big)\Big]}E_{in}(t). \quad (11)$$

The envelope of the optical field from cascaded two IM is periodic sinc-shape and it's a Nyquist pulse train in time domain. In the experiment, we can also adjust the phase difference of the two RF signals to ensure that the generated lines have the same phase.

Simulation results and discussions

Computer simulations by Virtual Photonics Inc. software package have been performed to study the performance of our proposed inc-shaped Nyquist pulse generation. A CW laser with a line width of 10 MHz at a frequency of 193.1 THz is sent to the cascaded intensity modulators. The IMs are with a same half-voltage of 5 V and a same extinction ratio of 30 dB. The frequency and amplitude of RF1 are 30 GHz and the frequency and amplitude of RF2 are 10 GHz, which satisfy the condition $f_2 = 3f_1$. The RF modulation index $m = 0.296$. We choose a DC voltage of IM1 to satisfy $\tan(\varphi) = +J_0(m)/J_1(m)$. In order to make the phase of optical frequency comb is linear-locked, the DC voltage of IM2 which we choose must satisfy $\tan(\varphi) = -J_0(m)/J_1(m)$.

An OFC with three equal tones, and 30 GHz frequency spacing can be generated as shown in Fig. 2a. The power variation is within 0.1 dB and the unwanted-mode suppression ratio is 40 dB, demonstrating a quasi-rectangular spectral shape, which can be utilized in the generation of Nyquist pulses. Figure 2b presents the simulated waveform of the three-tone OFC. The sinc pulse has a full-width at half maximum (FWHM) of 29.4 ps, and a repetition rate of 100 ps. When cascaded with another IM, 9 comb lines with nearly the

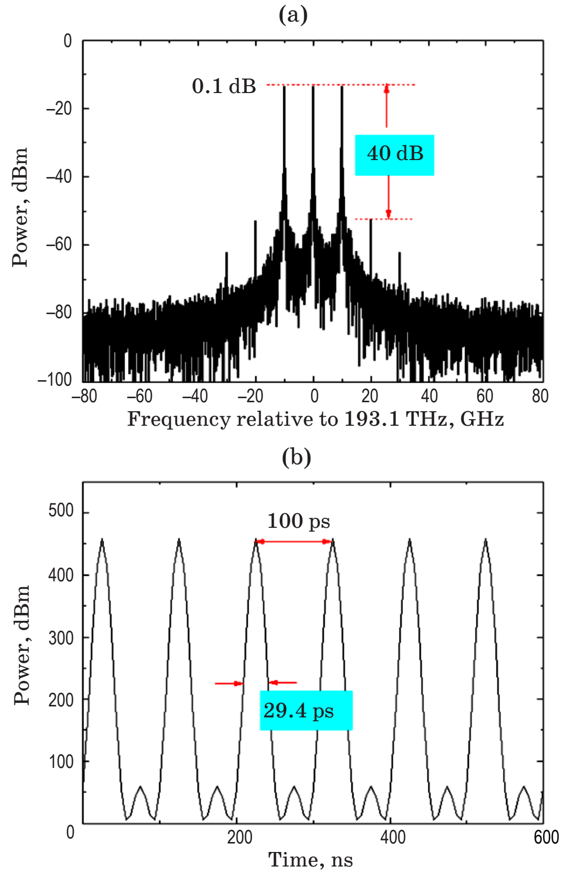

Fig. 2. Simulated output spectrum. (a) Optical spectrum and (b) time waveform of the generated Nyquist pulses by a single IM.

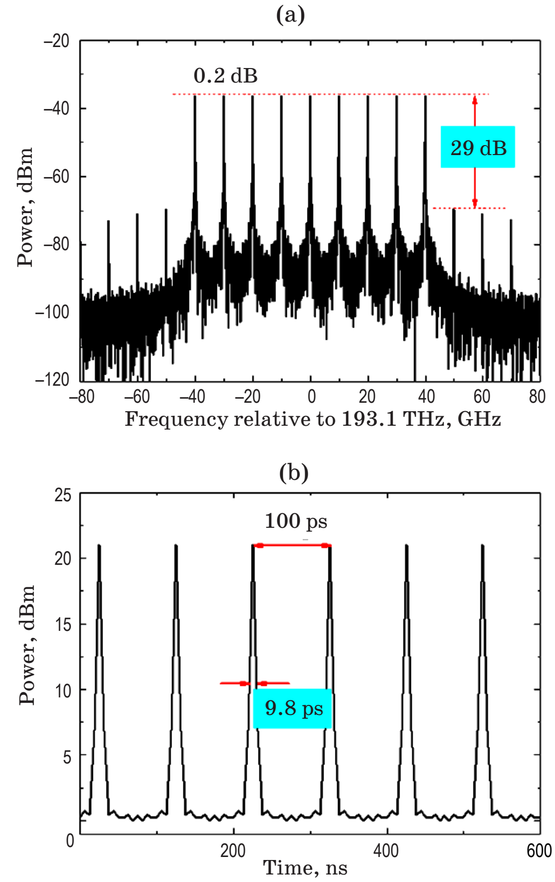

Fig. 3. Simulated output spectrum. (a) Optical spectrum and (b) time waveform of the generated Nyquist pulses by two cascaded IMs.

same amplitude are obtained and unwanted mode suppression ratio of 29 dB is achieved, which is shown in Fig. 3a. Figure 3b shows the waveforms of the generated Nyquist pulses. The Nyquist pulse has a FWHM duration of 9.8 ps and a repetition period of 100 ps. The pulse width and repetition rate can be changed by simply tuning the frequency comb parameters.

Conclusion

In this paper, a complete theoretical analysis of Nyquist pulse generation with 3 comb lines generated with a single intensity modulator is developed. It is proved that both the amplitude and phase of the sidebands in the optical frequency comb are important for generation of Nyquist pulses. Theoretically it is found that it is impossible to generate Nyquist pulses with more than 3 comb lines using a single IM even if 5 flat comb lines are realized. A Nyquist pulse can be generated with 9 comb lines by two cascaded intensity modulator, but the sum of the phase shift caused by the two DC voltages of the IM must be π.

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