DETERMINATION OF MAGNETO-OPTICAL CONSTANTS BY MEASURING KERR ROTATION ANGLES OF MAGNETIC FILMS WITH DIFFERENT SAMPLE STRUCTURES

© 2016. R. T. Zheng; X. W. Xu; X. A. Liang; Z. A. Lum

Data Storage Institute, A*STAR, Singapore

E-mail: ZHENG_Ruitao@dsi.a-star.edu.sg

A new data analysis method is proposed to determine magneto-optical (MO) constants by measuring Kerr rotation angles on the samples with different multilayer film structures. The MO film in the multilayer samples is protected by a SiO_2 film with two different thicknesses. An existing algorithm is used to calculate the Kerr rotation angle based on the MO theory developed in early 1990's. A new error function is defined to evaluate the distance from calculated Kerr angles to measured ones. The proposed method is developed based on Staged Continuous Tabu Search (SCTS) algorithm to retrieve the MO constant by searching the minimum of the error function. The uniqueness and accuracy of the MO constant determined by this method are discussed, and its relative error introduced by the measurement error of Kerr rotation angles is also analyzed. Without measuring the ellipticity, the proposed method is relatively simple and can be applied to determine the MO constants for the samples with two or three layers of films.

Keywords: *magneto-optical constant; Kerr rotation angle; multilayer film structure; data analysis.*

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Introduction

A growing interest in magneto-optical Kerr effect (MOKE) has resulted from its potential applications in magneto-optical (MO) spatial light modulator (SLM) driven by spin transfer switching, which is an essential device in 3D holographic display systems [1, 2]. The MO theory for multilayers was developed to investigate the MOKE in magnetic ultrathin films in early 1990's [3, 4]. In their theory and algorithm, the MO constant is one of the key parameters of magnetic films for MOKE simulation. Therefore, it is critical to determine the MO constants for developing new MO films for MO-SLM devices.

It is generally known that Kerr rotation angle and ellipticity should be measured in order to determine the MO constants based on the MOKE formula developed in Ref [3, 4]. However, it is inconvenient to measure the ellipticity [5–7], because either a photoelastic modulator or a quarter-wave plate is required [8, 9]. In comparison, the measurement of Kerr rotation angle is relatively simple. Therefore, the methods to determine the MO constants were proposed by measuring the Kerr rotation angles on the samples with a single MO film, without measuring the ellipticity [5–7]. In order to protect the MO film from oxidation, a thin protection layer was deposited on top of the MO film in Refs [10, 11]. However, both the Kerr rotation angle and ellipticity had to be measured in order to determine the MO constants for such a multilayer sample [11]. Recently more powerful vector-magneto-optical generalized ellipsometry (VMOGE) was developed to determine the MO constants for multilayer samples based on a data analysis method established [12]. However, the VMOGE setup was complicated because an octupole magnet was used to change magnetic field orientations during the measurements. Various data acquisition was required to measure a 4×4 Mueller matrix before the MO constant could be determined.

In this paper, we propose a new data analysis method to determine the MO constant for multilayer samples without measuring the ellipticity. An error function is defined to evaluate the errors between simulated Kerr angles and measured ones for film samples with a known multilayer structure. The method is developed based on a global optimization algorithm, i.e. Staged

Continous Tabu Search (SCTS) algorithm [13, 14] to determine the MO constants by searching the minimum of the error function. The proposed method is verified to be effective with simulation experiments, including the analysis of uniqueness and relative errors. Our method is relatively simple because only the Kerr rotation angles need to be measured with a simple optical setup at different wavelength.

Theoretical background

As an example of multilayer film stacks to present our method, we assume a beam of light is propagating from air into an MO film through a dielectric protection layer, with a normal incident angle, and the MO film is deposited on a substrate, as shown in fig. 1, the M matrix defined in Ref [3] can be written as

$$M = A_0^{-1}(A_1 D_1 A_1^{-1})(A_m D_m A_m^{-1})A_s. \tag{1}$$

Considering the right angle incidence, the matrices in Eq. (1) can be simplified as follows

$$A_j = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ iN_jQ_j/2 & -N_j & -iN_jQ_j/2 & -N_j \\ N_j & iN_jQ_j/2 & -N_j & iN_jQ_j/2 \end{bmatrix}, \tag{2}$$

$$D_j = \begin{bmatrix} 1/\exp(ikN_jd_j) & kN_jQ_jd_j/2\exp(ikN_jd_j) & 0 & 0 \\ -kN_jQ_jd_j/2\exp(ikN_jd_j) & 1/\exp(ikN_jd_j) & 0 & 0 \\ 0 & 0 & \exp(ikN_jd_j) & kN_jQ_jd_j\exp(ikN_jd_j)/2 \\ 0 & 0 & kN_jQ_jd_j\exp(ikN_jd_j)/2 & \exp(ikN_jd_j) \end{bmatrix}, \tag{3}$$

where N_j is the complex refractive index of the j^{th} layer of the stack, $k = 2\pi/\lambda$, λ – the wavelength of light, d_j – the film thickness of the j^{th} layer. Q_j is known as the MO constant of the j^{th} layer and it will be zero for all other layers except for MO film.

For the structure as shown in fig. 1, j is either 0, 1,..., m, or s. Here we define a matrix M_0 as follows

$$M_0 = A_0^{-1}A_1D_1A_1^{-1} = \begin{bmatrix} f_1+f_2 & 0 & 0 & (f_1-f_2)/N_1 \\ 0 & f_1+f_2 & (-f_1+f_2)/N_1 & 0 \\ f_3+f_4 & 0 & 0 & (f_3-f_4)/N_1 \\ 0 & -f_3-f_4 & (f_3-f_1)/N_1 & 0 \end{bmatrix}, \tag{4}$$

where

$$f_1 = (1+N_1)/4\exp(ikN_1d_1); \tag{5}$$

$$f_2 = \left[(1-N_1)\exp(ikN_1d_1)\right]/4; \tag{6}$$

$$f_3 = (1-N_1)/4\exp(ikN_1d_1); \tag{7}$$

$$f_4 = \left[(1+N_1)\exp(ikN_1d_1)\right]/4. \tag{8}$$

We can rewrite Eq. (1) in the form of Eq. (9) below,

$$M = M_0\left(A_mD_mA_m^{-1}\right)A_s. \tag{9}$$

For a specified film stack structure N_j and d_j can be measured, and hence M_0 and A_s can be calculated. From Eq. (9) we can see that the matrix M is determined only by A_m and D_m, which can be calculated if the value of Q_m is given.

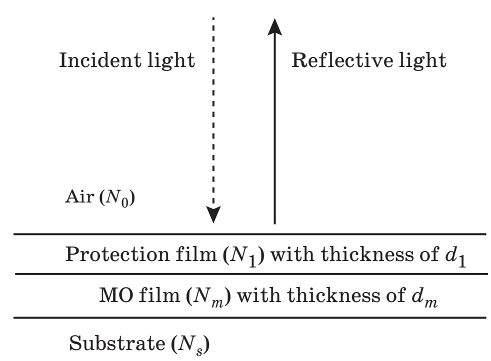


Fig. 1. Film stack structure of the sample for measurements. N_0, N_1, N_m and N_s are refractive index of air, protection film, MO film and substrate, respectively. d_1 and d_m are the thickness of protection film and MO film, respectively.

The M matrix can be expressed in 2×2 form and the Fresnel reflection coefficients for s- and p-lights can be obtained as follows [3]:

$$M = \begin{bmatrix} G & H \\ I & J \end{bmatrix}, \tag{10}$$

$$\begin{bmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{bmatrix} = IG^{-1}. \tag{11}$$

So, the Kerr rotation angle θ_k can be calculated by a complicated function $\kappa(Q_m)$ as follows

$$\theta_k = \text{real part}\left(r_{sp} / r_{ss}\right) = \kappa\left(Q_m\right). \tag{12}$$

The function $\kappa(Q_m)$ can be determined once all parameters such as N_0, N_1, d_1, N_m, d_m and N_s are specified. The Kerr rotation angle can be calculated if Q_m is given. Q_m has two variables, i.e. real part Q_m^r and imaginary part Q_m^i.

Data analysis method to determine the MO constant

Assuming the MO film with the same thickness has the same MO constant value, in our method we use two multilayer samples with the same MO film thickness and different thickness of the protection film, i.e. d_1' and d_2''. Considering the enhancement effect introduced by the protection film, different thickness of protection film will produce different Kerr rotation angles. We define an error function to evaluate the distance of calculated Kerr angles from measured ones as follows,

$$\text{error}\left(Q_m^r, Q_m^i\right) = \left(\kappa_1\left(Q_m^r, Q_m^i\right) - \theta_k'\right)^2 + \left(\kappa_2\left(Q_m^r, Q_m^i\right) - \theta_k''\right)^2, \tag{13}$$

where θ_k', θ_k'' are measured Kerr rotation angles with the protection film thickness of d_1' and d_1'' respectively, and κ_1 and κ_2 are functions defined by d_1' and d_1'' respectively. If we can find a suitable value of Q_m which provides the minimum error value defined in Eq. (13), we will be able to determine the MO constant of the MO film. In order to search the minimum error, we have developed a new data analysis method based on the SCTS algorithm, and the flowchart of the proposed method is shown in fig. 2.

As shown in fig. 2, the Kerr rotation angles are measured on two multilayer samples with different thickness of protection films and input into the program. At first, the program will randomly select an initial solution of Q_m as a starting point of the data analysis process. The Kerr rotation angles are calculated with this initial solution based on the known parameters of the samples. Corresponding error value is then calculated by Eq. (13). After that the SCTS algorithm will search the minimum of error value. In next iteration, the initial solution is replaced by the Q_m that produced the minimum error value. We define two criteria for terminating the program as follows,

1) a specified number of iteration is achieved,

2) a specified error value is obtained.

If one of the above criteria is met, the program will stop and output the updated initial solution as the final solution of Q_m. Otherwise, the program will continue to use the updated initial solution to search the minimum error value until meeting one of the above criteria.

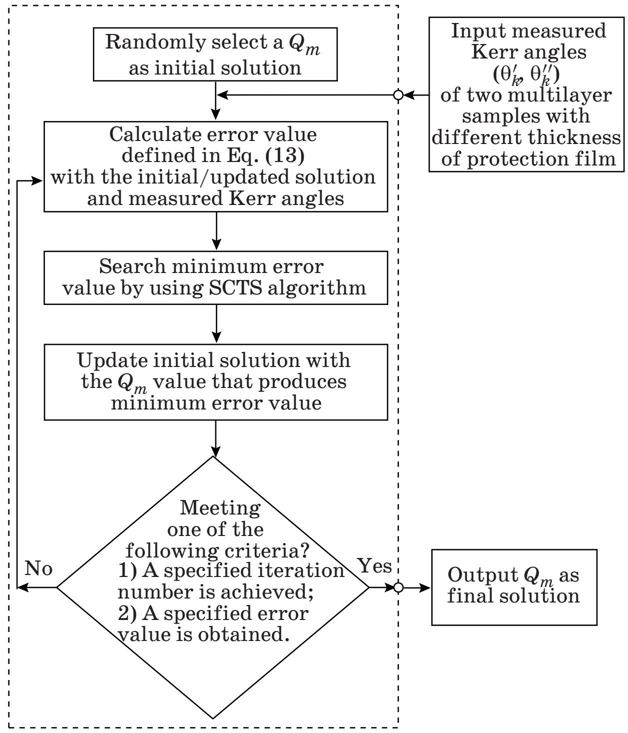


Fig. 2. Flowchart of the proposed method.

Simulation experiments and discussions

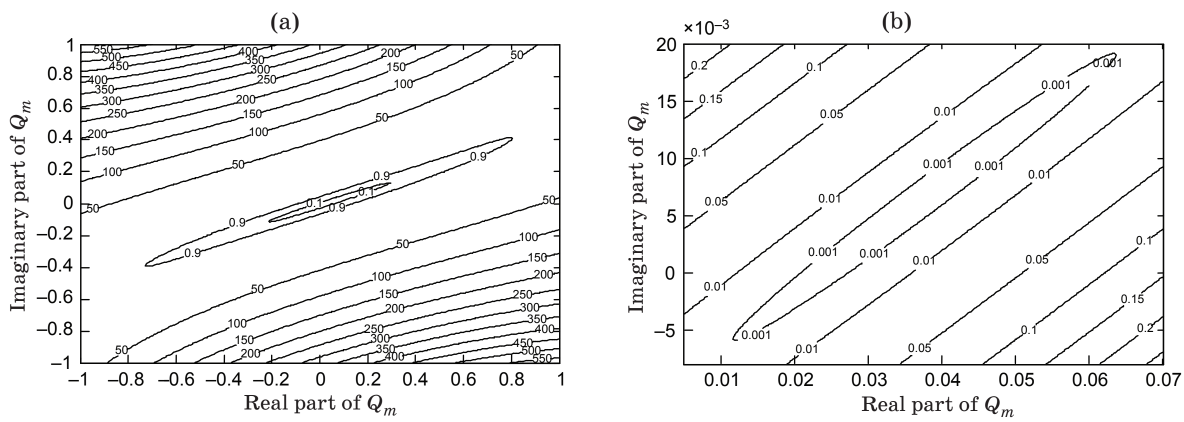
To verify the effectiveness of the proposed method, the Fe film with a known MO constant (Q_m = 0.0376 + i0.0066) [3] is used as an example of MO film to simulate the experiments for data analysis. The wavelength used in simulation is 632.8 nm. In this example, two different multilayer film stack samples are used to determine the MO constant. The first sample is prepared by depositing a 10 nm thick Fe film on silicon substrate and a 5 nm thick SiO_2 film as a protection layer on top of the MO film to protect it from oxidation. The second sample is prepared in the same way except that the thickness of SiO_2 film is changed from 5 nm to 102.6 nm, which corresponds to an optical thickness of one quarter of wavelength (i.e. $N_1 d_1 = \lambda/4$), because the maximum Kerr rotation enhancement introduced by the layer of SiO_2 is expected to occur at this thickness [15]. The refractive index values obtained from Ref [16] and thickness of all the films in these two samples are listed in the table.

Based on the parameters listed in the table and the known MO constant of Fe film, the Kerr rotation angles of the above two samples are calculated to be $\theta_k = 0.031°$ and $0.198°$ for the first and second samples respectively, by using Eq. (12). In our simulation experiments, the above two calculated Kerr rotation angles are assumed to be the "measured" ones, which are then used as input values to determine the MO constant (Q_m) of Fe film with our proposed method as shown in fig. 2.

Figure 3a shows the dependence of error value distribution on real and imaginary parts of the MO constant Q_m. Error values at different levels are numerically calculated by Eq. (13), and marked on the contour map in fig. 3a. It can be seen that the minimum error value is located inside the circle area of 0.1. Figure 3b shows the magnified contour map of fig. 3a, from which the minimum error value can be found inside the circle area of 0.001. Figure 3c is the contour map further magnified from fig. 3b, and the minimum error value is found to be at $Q_m = 0.0376 + i0.0066$, which is the same as the known Q_m value of Fe film. It indicates that the final solution (Q_m) of Eq. (13) output from the proposed method in fig. 2 produces a unique minimum error value. To verify the reliability of the proposed method, we use different randomly selected starting points (i.e. initial solution of Q_m) to search the minimum

Refractive index and thickness of all the films in two samples

No. of sample	N_s	N_1	d_1, nm	N_m	d_m, nm
1	3.8827+0.019626i	1.5426	5	2.895+3.0688i	10
2	3.8827+0.019626i	1.5426	102.6	2.895+3.0688i	10



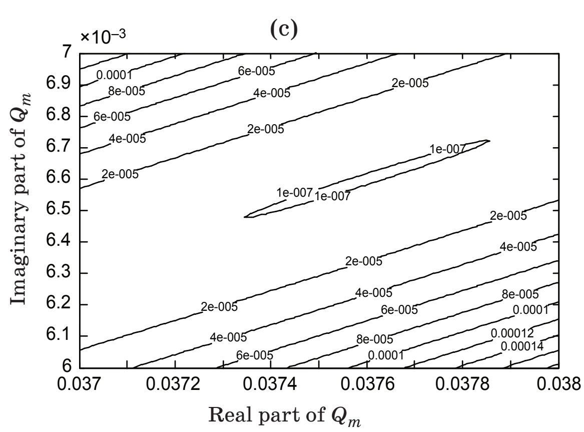


Fig. 3. (a) – Error value distribution contour map *vs* Q_m over [−1, 1]. (b) – Magnified contour map of (a), showing minimum point can be found within the circle area of 0.001. (c) – Further magnified contour map of (b). The minimum point appears at the location of 0.0376 of real part and 0.0066 of imaginary part.

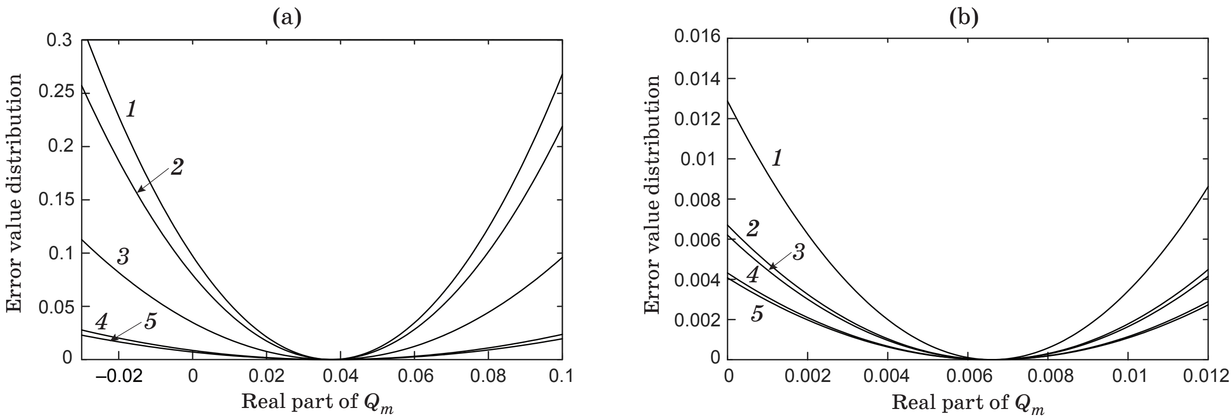


Fig. 4. a – Error value distribution *vs* real part of Q_m when imaginary part of Q_m is fixed at 0.0066; b – Error value distribution *vs* imaginary part of Q_m when real part of Q_m is fixed at 0.0376. Lines *1, 2, 3, 4* and *5* are calculated error values for the sample pairs with the SiO_2 film thickness of 5 nm/102.6 nm, 5 nm/128.2 nm, 5 nm/153.9 nm, 5 nm/25.6 nm and 5 nm/51.3 nm, respectively.

error value of Eq. (13) for 100 times and the results show that the same MO constant can be determined by the proposed method with 100% successful rate.

In fig. 4 we investigate the error value distribution vs Q_m by changing the thickness of SiO_2 film for the second sample. In this study, the thickness of SiO_2 film is set as 5 nm for the first sample, and 25.6 nm, 51.3 nm, 102.6 nm, 128.2 nm and 153.9 nm for the second sample, corresponding to the optical thickness of λ/16, λ/8, λ/4, 5λ/16 and 3λ/8, respectively. At different SiO_2 film thickness of the second sample, the error value distributions are plotted versus the real part of Q_m by fixing the imaginary part of Q_m at 0.0066 in fig. 4a, and versus the imaginary part of Q_m by fixing the real part of Q_m at 0.0376 in fig. 4b. Usually, the steeper the distribution is, the more accurate the Q_m can be determined with a specified error value set in the criteria for terminating the program as shown in fig 2. From fig. 4 one can see that the sample pair with 5 nm/102.6 nm-thick SiO_2 films produces the steepest error value distribution vs Q_m. Therefore the SiO_2 film thickness of 5 nm/102.6 nm is preferred for sample preparation.

The relative error of the MO constant determined by the proposed method is also investigated. We purposely introduce different relative errors into the 'measured' Kerr rotation angles by deviating them from the calculated ones. The relative error of the MO constant is calculated by comparing the output Q_m of final solution in fig. 2 with the reported Q_m. Figure 5 shows the dependence of the relative errors of Q_m on the relative errors in Kerr rotation angles. It is noted that

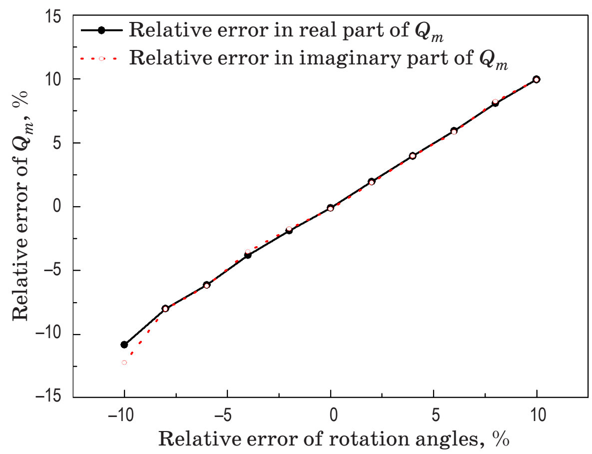


Fig. 5. Relative errors of Q_m vs the relative errors in Kerr rotation angles.

the relative error in the measured Kerr rotation angles should be smaller than 5% in order to ensure a relative error in MO constant less than 5%.

The simulation experiments presented above is based on a two-layer film stack structure (i.e. protection film and MO film) as shown in fig. 1. Because the matrix method [3] used to derive Eq. (1) can also be used for a structure with more than two layers of films, the effectiveness of the proposed method has also been evaluated by using a three-layer film stack structure. One more 5 nm-thick SiO_2 film is inserted between the MO film and substrate. The same simulation experiments are repeated. The resulted error value distribution contour maps and measurement error analysis show similar characteristics presented in fig. 3 and fig. 5, respectively. It means that the proposed method is also valid for the determination of MO constant for the samples with three layers of films. It is important for some special fabrication conditions required by the samples such as the MO film of CoFeB that needs to be sandwiched by Ta and MgO films in order to maintain perpendicular magnetic anisotropy (PMA) [17].

Conclusion

A new data analysis method has been developed to determine the MO constant. With this method, only the Kerr rotation angles need to be measured on two samples with different thickness of SiO_2 film. Without measuring the ellipticity the MO constant can be retrieved by searching the minimum of the error function with the SCTS algorithm. The presented method has shown the uniqueness of the output MO constant. The sample pair with 5 nm and 102.6 nm thick SiO_2 films is recommended to improve the accuracy of the MO constant. The relative error in the measured Kerr rotation angles should be smaller than 5% in order to ensure a relative error in MO constant less than 5%. The proposed method is relatively simple and can be extended to determine the MO constants for the samples with three layers of films.

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