Оптический журнал
Иконика

DOI:10.17586/1023-5086-2022-89-02-43-51

УДК 535.8

New topological charge parallel measurement method of optical vortex based on computer-generated holography

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Annotation

The generation, propagation, and application of optical vortex have been major researched topics in recent years. In this study, a parallel measurement approach is presented to analyze the topological charge of single and composite optical vortices with the help of the spatial light modulator. The topological charges of single and composite optical vortices are measured by observing the Gaussian point position of the diffracted field, which solves the difficult problem of measuring the composite optical vortex and the optical vortex with large topological charge. The presented method is feasible and has strong expansibility. More composite optical vortex can be measured easily by using different computer-generated holography. When the optical vortex is measured correctly, it can present a Gaussian point, which retains the characteristics of optical vortex. Therefore, this point will be important in the optical vortex communication.

Keywords: optical vortex, parallel measurement, computer-generated hologram

Acknowledges. This work was funded by the National Natural Science Foundation of China (NSFC) (61475098). This work was supported by 111 Project (D20031).

For citation: Xianpeng Liu, Sujuan Huang, Wancai Xie, Zhonghua Pei. New topological charge parallel measurement method of optical vortex based on computer-generated holography // Оптический журнал. 2022. Т. 89. № 2. С. 43–51. DOI: 10.17586/1023-5086-2022-89-02-43-51

OCIS codes: 090.1995, 200.4740.

Новый метод параллельного измерения топологического заряда в оптических вихрях на основе компьютерной голографии

Xianpeng Liu, Sujuan Huang, Wancai Xie, Zhonghua Pei

Аннотация

Генерация, распространение и применение оптических вихревых пучков являются популярными предметами исследований в последние годы. В данной работе рассмотрена возможность анализа топологического заряда одиночных и составных оптических вихрей с использованием параллельных измерений с помощью компьютерной голограммы, реализуемой пространственными модуляторами света. Топологические заряды одиночных и составных оптических вихрей измеряются путём наблюдения положения гауссовской точки, возникающей при дифракции измеряемого пучка на соответствующей компьютерной голограмме. Это позволяет решить сложную задачу измерения таких вихрей, обладающих большим топологическим зарядом. Представленный метод достаточно прост в реализации и допускает дальнейшие расширения путём

генерирования более сложных измерительных голограмм, соответствующих более сложным конфигурациям вихревых полей. Распределение поля в гауссовской точке, соответствующей данному оптическому вихрю, сохраняет его характеристики, что может оказаться полезным в коммуникационных задачах, использующих вихревые свойства оптического излучения.

Ключевые слова: оптический вихрь, параллельное измерение, компьютерная голограмма

1. INTRODUCTION

The optical vortex has a helical wavefront phase structure, and its phase factor is described as $\exp(il\varphi)$, where l is the number of the topological charges and a theoretically desirable arbitrary integer, and φ is the azimuth angle. The optical vortex intensity shows a dark hollow envelope structure due to the presence of its phase singularity [1–3]. In recent years, the optical vortex has received wide attention due to the unique character of its orbital angular momentum. Optical vortex has made important achievements in the fields of optical communication, microparticle control, quantum communication, and quantum entanglement [4–7]. However, the wide application of optical vortex is inseparable from the correct measurement of its orbital angular momentum.

Many common methods, including the interference, diffraction, and computer-generated hologram (CGH) methods, can be used to measure the topological charge of optical vortex [8–10]. The common interferometry is Mach–Zehnder interferometry, Sagnac interferometry, and lateral shearing interferometry [11–13]; the diffraction method includes single-slit interference, double-slit interference, and triangular hole interference [14, 15]; the CGH method includes gradient and ring gratings [16, 17]. The optical vortex is measured in these methods by counting the number of fringes in the light field. Therefore, the measurement method above is not ideal for the multi-composite optical vortex and the optical vortex with large topological charge.

In this study, the object-oriented CGH by conjugate-symmetric extension of Fourier transform is proposed to measure single optical and composite optical vortices [18, 19]. Compared with other CGHs, the proposed method not only can measure the topological charge of single and composite optical vortices but also shows a good measurement effect on the optical vortex with large topological charge. The proposed measurement method of optical vortex depends on the position of the light field rather than the number of the light field. In addition, the presented experiment is feasible and has strong expansibility. More composite optical vortex can be measured by switching the CGH. The correct measurement result of the light field appears as a Gaussian point, which can be used in the optical vortex communication.

2. THEORETICAL ANALYSIS

2.1. Composite optical vortex

In this study, the vortex beam of Laguerre–Gaussian (LG) is used in the laboratory because of its circular symmetry and ready availability. A typical LG vortex beam commonly has p radial nodes. We take $p = 0$ in the experiment for simplicity. Therefore, the optical vortex shows a dark hollow structure, and its complex amplitude can be expressed as

$$E_0{}^l = \frac{1}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|}\exp\left(\frac{-r^2}{w(z)^2}\right)\times \\ \times\exp\left[\mathrm{i}\left(kz-\frac{kr^2}{2R}+\varphi\right)\right]\exp(-il\theta), \quad (1)$$

where r, θ, and z are the cylindrical coordinates,

$$w(z) = w_0\sqrt{1+(z/z_R)^2} \quad (2)$$

denotes the transverse beam size at the propagation distance z, w_0 is the waist radius, l is the number of topological charges, k is the wave number, z_R represents the Rayleigh range of the Gaussian envelope, and R is the curvature radius of the wavefront, the Gouy phase φ is given by

$$\varphi = (|l|+1)\arctan(z/z_R). \quad (3)$$

Therefore, the complex amplitude expression of the composite optical vortex, which is the su-

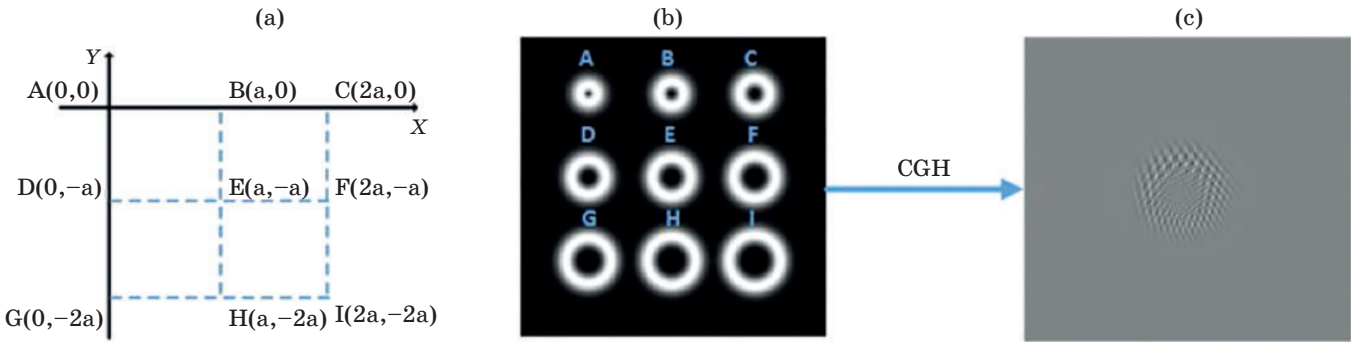


Fig. 1. Conjugate symmetric extension of Fourier computer-generated holography. Plane distribution diagram of optical vortex array (a), theoretical intensity distribution of optical vortex array (b), CGH of optical vortex array (c).

perposition of two collinear LG vortex beams of topological charge l_1 and l_2, is as follows:

$$E_c = E^{l_1} + E^{l_2}, \tag{4}$$

where l_1 and l_2 are optical vortices with different topological charges.

2.2. Optical vortex array

The following Formula (5) can be obtained by changing the cylindrical coordinate system of Formula (1) to Cartesian coordinates.

$$E_0{}^l = \frac{1}{w(z)}\left(\frac{\sqrt{x^2+y^2}\sqrt{2}}{w(z)}\right)^{|l|}\exp\left(\frac{-x^2-y^2}{w(z)^2}\right)\times \\ \times\exp\left[\mathrm{i}\left(kz-\frac{k\left(x^2+y^2\right)}{2R}+\varphi\right)\right]\cdot\exp(-il\theta), \tag{5}$$

where r is given by

$$r = \sqrt{x^2+y^2}. \tag{6}$$

As shown in Fig. 1a, the plane distribution diagram of 3×3 optical vortex array consists of nine parts: A (0, 0), B (a, 0), C (2a, 0), D0 (a, −a), F (2a, −a), G (0, −2a), H (a, −2a), and I (2a, −2a). The nine parts are different optical vortices with different topological charges. Then, the 3×3 optical vortex array in Cartesian coordinates is expressed as follows:

$$E_{array} = E^{l_A} + E^{l_B}(x-a,y) + E^{l_C}(x-2a,y) + \\ + E^{l_D}(x,y+a) + \ldots + E^{l_I}(x-2a,y+2a). \tag{7}$$

2.3. Conjugate symmetric extension of Fourier computer-generated holography

Optical vortex array complex amplitude distribution $f_0(m, n)$ can be simply expressed as

$$f_0(m,n) = A(m,n)\exp\left[\mathrm{i}\varphi(m,n)\right] \\ m = 1, 2, \cdots, M/2-1; n=1, 2, \cdots, N-1, \tag{8}$$

where $A(m, n)$ and $\varphi(m, n)$ represent the amplitude and the phase of the optical vortex array, respectively. In this method, a complex object light $f_0(m, n)$ is conjugate-symmetrically extended into the form expressed as below with the assumption that M and N are both even.

$$f(m,n) = \begin{cases} f_0(m,n) & m = 1, 2, \cdots, M/2-1; n = 1, 2, \cdots, N-1 \\ f_0^*(M-m, N-n) & m = M/2+1, \cdots, M-1; n = 1, 2, \cdots, N-1. \\ 0 & m = 0 \text{ or } n = 0 \text{ or } m = M/2 \end{cases} \tag{9}$$

The superscript "*" indicates complex conjugate. Let the $f(0, n) = f(m, 0) = f(M/2, 0) = 0$, and the center of symmetry is $(M/2, N/2)$.

Two-dimensional discrete Fourier transform of $f(m, n)$

$$F(\mu,\nu) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\exp\left[-\mathrm{i}2\pi\left(\frac{m\mu}{M}+\frac{n\nu}{N}\right)\right] \quad \begin{matrix}\mu = 0, 1, \ldots, M-1 \\ \nu = 0, 1, \ldots, N-1\end{matrix}, \tag{10}$$

where μ and ν are the frequency domain sample numbers in horizontal and vertical directions, respectively. By expressing the optical vortex array in the form $f_0(m, n)$, we can obtain the hologram of the original light wave as follows:

$$F(\mu,\nu)=\frac{2}{MN}\sum_{m=1}^{(M/2)-1}\sum_{n=1}^{N-1}\left\{A(m,n)\cos\left[2\pi\left(\frac{m\mu}{M}+\frac{n\nu}{N}\right)-\varphi(m,n)\right]\right\}\quad\begin{matrix}\mu=0,1,\ldots,M-1\\ \nu=0,1,\ldots,N-1\end{matrix}. \tag{11}$$

Formula (11) contains the amplitude information $A(m, n)$ of the object light wave and the phase information $\varphi(m, n)$ of the object light wave and is a real value function. This real-valued function $F(\mu, \nu)$ is then linearly mapped to a range of [0, 255] and coded into a gray scale hologram, as the CGH shown in Fig. 1c.

2.4. Measurement theory

The incident light field contains several topological charges, such as composite optical vortex. Thus, the CGH of 3×3 optical vortex arrays can be used to measure the incident light. The expression is given below:

$$\begin{aligned}E=E_{array}*E_c=&(E^{l_A}+E^{l_B}(x-a,y)+\\&+E^{l_C}(x-2a,y)+E^{l_D}(x,y+a)+\\&\ldots+E^{l_I}(x-2a,y+2a))*(E^{l_1}+E^{l_2})=E^{l_A+l_1}+\\&+E^{l_A+l_2}+(E^{l_B+l_1}+E^{l_B+l_2})(x-a,y)+\\&+(E^{l_C+l_1}+E^{l_C+l_2})(x-2a,y)+\ldots\\&+(E^{l_I+l_1}+E^{l_I+l_2})(x-2a,y+2a)\end{aligned} \tag{12}$$

If the 3×3 optical vortex array contains the same topological charges as the incident light field, then the Gaussian point will show at the corresponding position. Therefore, the topological charges of incident light field can be measured by the position of Gaussian point at the diffraction light field.

In this study, conjugate symmetric extension of Fourier computer-generated holography is used. Its essence is conjugate symmetric complex function of Fourier transform, which is a real function. The 3×3 optical vortex array (1–9 topological charge) as the complex amplitude of the object light wave, as shown in Fig. 1b, is conjugate-symmetrically extended. Then, the discrete Fourier transform is taken. The real value function contains amplitude information and phase information of the object light wave. Finally, it is appropriately encoded to a gray scale hologram. The CGH is loaded onto a spatial light modulator (SLM) to reproduce the original object light wave, which is a 3×3 optical vortex array, as the incident light is a Gaussian spot. When the incident light is optical or composite optical vortex, the orbital angular momentum contained in the incident light field can be measured at the corresponding position. Determining the orbital angular momentum of the incident light field by using the position of the Gaussian point rather than the fringes in the light field is more accurate and intuitive. Therefore, the measurement of optical vortex with large topological charge is more effective because its fringes are too many to count. If the incident light field contains a topological charge greater than 9, then we can switch the different CGHs, which are loaded on the SLM to measure the incident light field correctly. The CGH of 3×3 optical vortex arrays can correctly measure nine kinds of topological charges.

3. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 2a. The laser beam emitted by the broadband polarization maintaining laser (λ = 1550 nm) passes through the polarization controller (Pol) to make

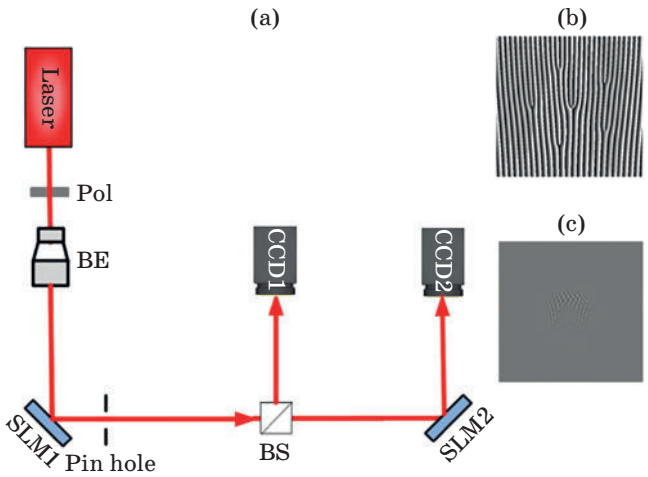


Fig. 2. Experimental setup diagram (a), phase map that will be loaded on the SLM1 (b), CGH map that will be loaded on the SLM2 (c).

its polarization direction suitable for the reflection-type phase space optical modulation (SLM1, Holoeye, PLUTO, pixels number 1920×1080). The beam expander system is used to obtain the appropriate Gaussian light projected on the first SLM (denoted as SLM1), and the composite optical vortex flare forked phase diagram is loaded on the SLM1 as shown in Fig. 2b. The generated composite optical vortex is selected via a pinhole and divided into two parts by the beam splitter. One beam part is recorded by CCD1 (charge coupled device) with a resolution of 1280×1024 pixels, and the other beam is projected onto the second SLM (SLM2), which is the same as SLM1. When the CGH as shown in Fig. 2c is loaded on the SLM2, the incident composite optical vortex can be measured through the light field recorded by CCD2.

4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1. Single optical vortex measurement

In this section, the 3×3 optical vortex array ($l = 1–9$) CGH is loaded on the SLM2 to measure nine different optical vortices ($l = 1–9$). When flare forked phase diagram of single optical vortex is loaded on the SLM1, the generated single optical vortex can be recorded by the CCD1. When the specific CGH (Fig. 1c) is loaded on the SLM2, the orbital angular momentum of the incident single optical vortex can be measured.

As shown in Fig. 3, nine light fields are recorded on the CCD2 when flare forked phase diagram ($l = 1–9$) is loaded on the SLM1. When the topological charge of incident optical vortex

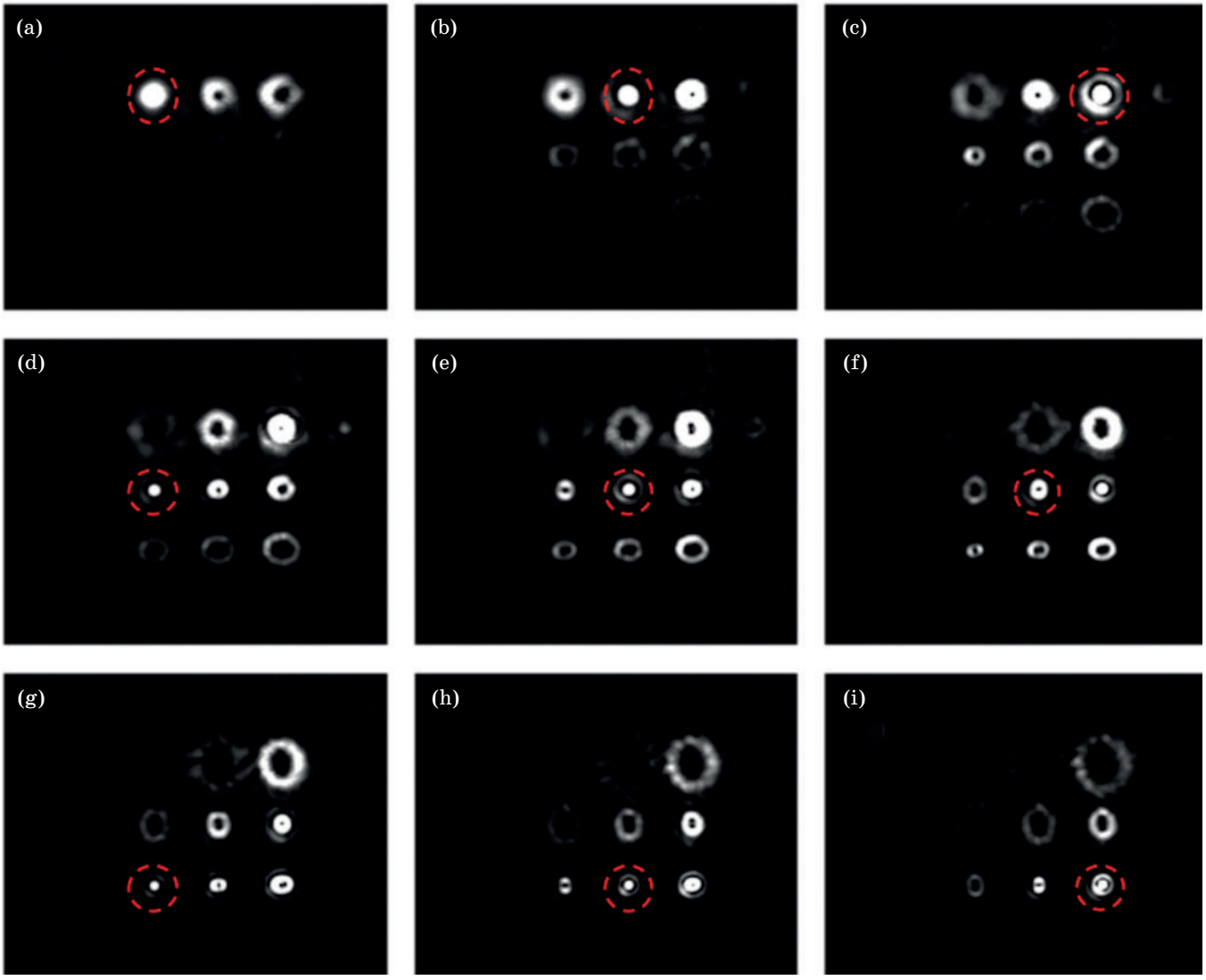


Fig. 3. Measurement diagram of single optical vortex (a)–(i) $l = 1–9$.

is equal to 1, the Gaussian point is generated at the first position, and the position marked with a red dashed circle is used as the reference. When the topological charge of incident optical vortex is equal to 2, the Gaussian point is generated at the second position, and so on. Therefore, we can determine whether the incident light field contains the special optical vortex component of the status of each position.

4.2. Composite optical vortex measurement

The experiment above has achieved good results for the measurement of the single optical vortex. The proposed method based on conjugate-symmetric extension of Fourier transform holography not only has a good effect on the measurement of a single light vortex but also has a good performance for the measurement of composite optical vortex. The composite optical vortex is generated by loading the composite phase diagram on SLM1. In this study, the flare forked phase diagram of composite optical vortex is used to improve the efficiency of generating composite optical vortex for achieving a good generation effect [20–21].

As shown in Fig. 4, three different flare forked phase diagrams of composite optical vortex are loaded on SLM1, and the corresponding three composite optical vortices are recorded by CCD1. The optical field recorded by the CCD 1 can judge whether the composite optical vortex is generated correctly. The 3×3 optical vortex array (l = 1–9) CGH is loaded on the SLM2, and the light fields recorded by CCD2 are shown in Fig. 5. When the incident light is a composite optical vortex composed of 2, 7, the detection light field at the position of 2 and 7 presents the Gaussian point, which means the composite optical vortex has the topological charge of 2 and 7. Similar to the measurement of single optical vortex, we also can determine wheth-

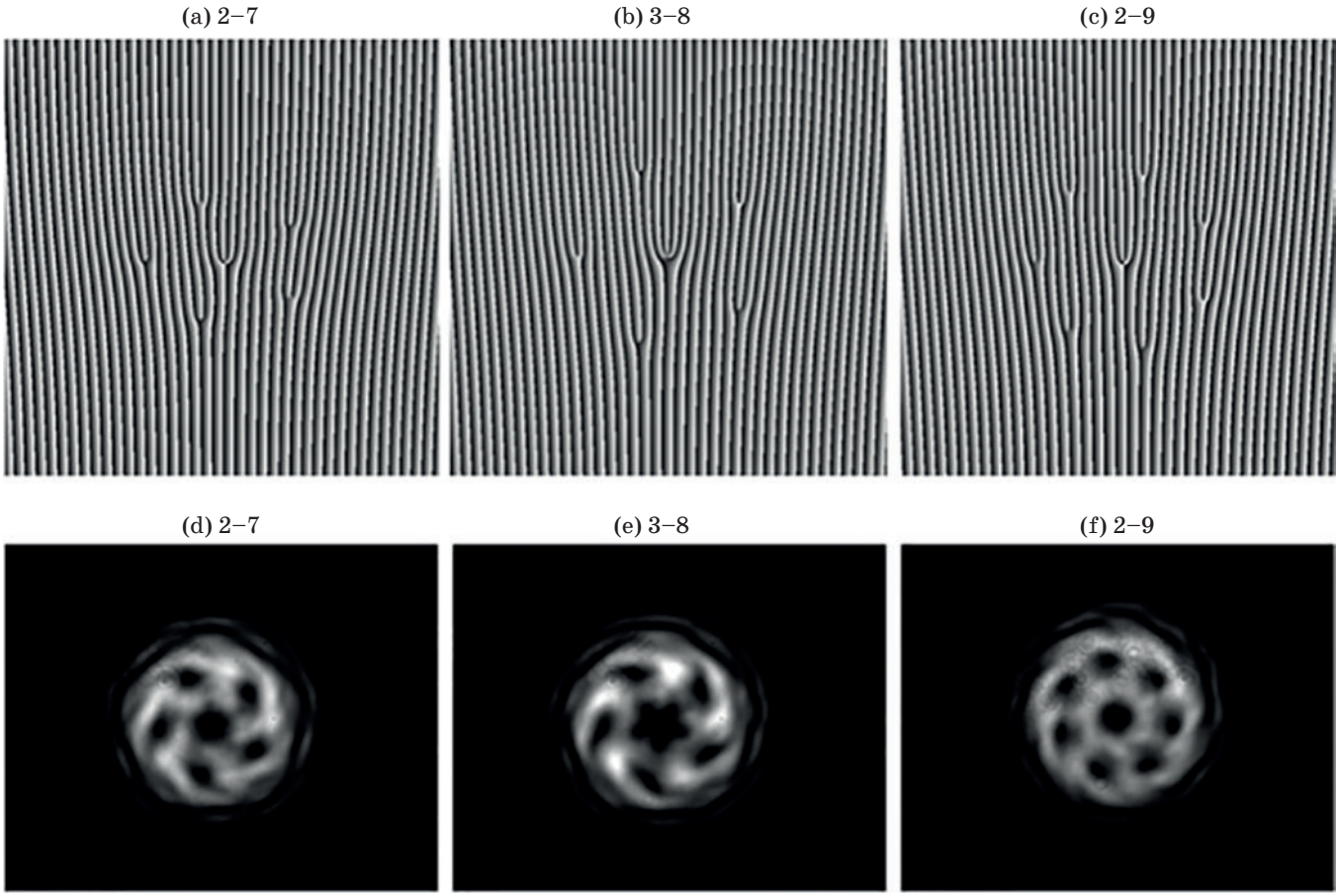


Fig. 4. Generation of composite optical vortex. Hologram that will be loaded on the SLM (a)–(c), intensity distribution with $l_1 = 2$ and $l_2 = 7$ (d), (e) intensity distribution with $l_1 = 3$ and $l_2 = 8$, intensity distribution with $l_1 = 2$ and $l_2 = 9$ (f).

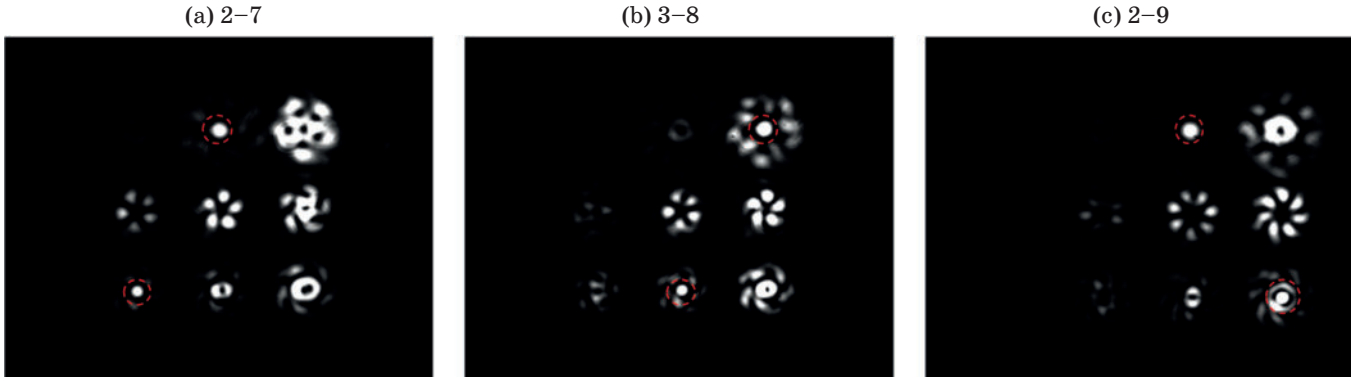


Fig. 5. Measurement diagram of composite optical vortex. Composite optical vortex measurement with $l_1 = 2$ and $l_2 = 7$ (a), composite optical vortex measurement with $l_1 = 3$ and $l_2 = 8$ (b), composite optical vortex measurement with $l_1 = 2$ and $l_2 = 9$ (c).

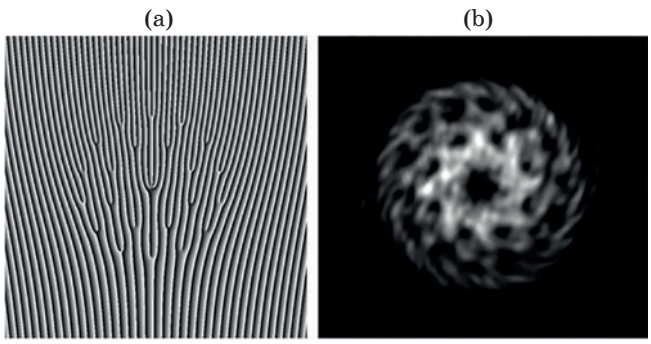


Fig. 6. Generation of composite optical vortex. Hologram that will be loaded on the SLM (a), intensity distribution with $l_1 = 3$, $l_2 = 11$, and $l_3 = 25$ (b).

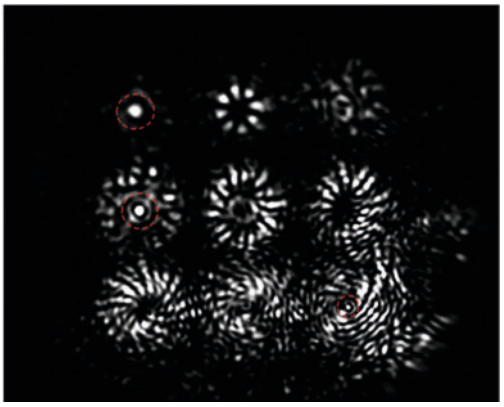


Fig. 7. Measurement of composite optical vortex and optical vortex with large topological charge.

er the incident light field contains the special optical vortex component of the status of each position. Experiment result shows that the proposed method is effective and feasible for the measurement of composite optical vortices.

The topological charge (TC) of optical vortex is measured by the position of the diffracted light field, which is different from other CGH measurement methods. Therefore, this strategy is more effective for the measurement of optical vortex with large topological charge and multi-composite optical vortex. As shown in Fig. 6, the flare forked phase diagrams of composite optical vortex composed of 3, 11, 25 are loaded on SLM1 in Fig. 6a, and the corresponding three composite optical vortices are recorded by CCD1 in Fig. 6b. Then, the CGH of 3×3 optical vortex array that consists of 3, 6, 9, 11, 14, 17, 20, 22, 25 is loaded on the SLM2. Thus, we can obtain the diffracted light field as shown in Fig. 7. Three positions show Gaussian point corresponding to 3, 11, 25, which shows the incident light field consisting of the topological charge of 3, 11, 25. Therefore, the proposed method greatly affects the measurement of multi-composite optical vortices.

5. CONCLUSION

In this study, the object-oriented CGH by conjugate-symmetric extension of Fourier transform is proposed to measure single and composite optical vortices. This method shows a good measurement effect. By using the position judgment method, the large TC and composite optical vortices can be measured. In addition, the light field measured by this experiment shows a Gaussian point, which can be further applied to the application of optical vortex

communication. This point is important for the application of the composite optical vortex. The further direction of this experiment is to increase the number of arrays to 5×5 or even 10×10 in one hologram. Thus, the number of optical vortices to be measured at the same time will be further increased. Our results may be useful in some fields of optical vortices, such as quantum information processing and optical communication.

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Submitted 13.01.2021, approved after reviewing 03.04.2021, accepted for publication 09.12.2021